

Precision Corrections to Supersymmetric Unification

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Abstract

We compute the full set of weak-scale gauge and Yukawa threshold corrections in the minimal supersymmetric standard model, including all finite (non-logarithmic) corrections, which we show to be important. We use our results to examine the effects of unification-scale threshold corrections in the minimal and missing-doublet SU(5) models. We work in the context of a unified mass spectrum, with scalar mass M_0 and gaugino mass $M_{1/2}$, and find that in minimal SU(5) with squark masses less than one TeV, successful gauge and Yukawa coupling unification requires $M_{1/2} \ll M_0$ and $M_0 \simeq 1$ TeV. In contrast, we find that the missing-doublet model permits gauge and Yukawa unification for a wide range of supersymmetric masses.

1 Introduction

With the advent of precision measurements at LEP and the observation that the gauge couplings unify in the minimal supersymmetric standard model [1], there has been a resurgence of interest in supersymmetric grand unified theories. In most analyses the values of the electromagnetic coupling, α_{EM} , and the standard-model weak mixing angle, $s_{\text{SM}}^2 \equiv \sin^2 \theta_{\text{SM}}(M_Z)$, are taken from experiment, and converted into supersymmetric \overline{DR} parameters, $\hat{\alpha}$ and $\hat{s}^2 \equiv \sin^2 \hat{\theta}(M_Z)$, using the leading-logarithmic contribution to the supersymmetric threshold corrections. The \overline{DR} parameters are then used to determine the \overline{MS} strong coupling constant at the Z -scale, $\alpha_s(M_Z)$, ignoring all unification-scale threshold corrections. Alternatively, the measured value of $\alpha_s(M_Z)$ is used to constrain the unification-scale parameter space.

In this letter we will take a closer look at this procedure, and report on the results of a complete next-to-leading-order analysis of supersymmetric unification. Our approach is new in two respects. First, we include all finite corrections. This implies that our weak mixing angle \hat{s}^2 is related to s_{SM}^2 as follows,

$$\hat{s}^2 = s_{\text{SM}}^2 + \text{leading log} + \text{finite}, \quad (1)$$

where the leading logarithms are of the form $\log(M_{\text{SUSY}}/M_Z)$ and the finite corrections are of order M_Z^2/M_{SUSY}^2 . If all the superpartner masses are heavier than a few times M_Z , the finite corrections are negligible, in accord with the decoupling theorem. However, realistic supersymmetric models typically have light particles with masses near M_Z , so the finite corrections can be significant.

Second, we do not use a combined-fit value of s_{SM}^2 to compute \hat{s}^2 because the finite term in eq. (1) is different for each observable. Therefore, in our analysis, we calculate \hat{s}^2 directly from a given set of inputs, namely, the Fermi constant G_F , the Z -boson mass M_Z , the electromagnetic coupling α_{EM} , the top-quark mass m_t , and the parameters that describe the supersymmetric model. (See also ref. [2].)

We emphasize that a precise evaluation of \hat{s}^2 is essential because the renormalization group equations (RGE's) can naturally amplify small corrections. For example, the one-loop renormalization group equations, $d\hat{g}_i/dt = \beta_i \hat{g}_i^3/16\pi^2$, together with gauge coupling unification, imply

$$\frac{\beta_2 - \beta_3}{\hat{g}_1^2(\mu)} + \frac{\beta_3 - \beta_1}{\hat{g}_2^2(\mu)} + \frac{\beta_1 - \beta_2}{\hat{g}_3^2(\mu)} = 0, \quad (2)$$

at any scale μ . Solving for $\alpha_s(M_Z)$ and varying the inputs $\hat{\alpha}$ and \hat{s}^2 , we find

$$\frac{\delta\alpha_s}{\alpha_s(M_Z)} \simeq \frac{\delta\hat{\alpha}}{\hat{\alpha}} - 7.5 \frac{\delta\hat{s}^2}{\hat{s}^2}. \quad (3)$$

This shows that a 1% error in the determination of \hat{s}^2 leads to an error in the evaluation of $\alpha_s(M_Z)$ of 7.5%.

In what follows, we will briefly describe our calculation of $\hat{\alpha}$ and \hat{s}^2 . We will then examine our prediction for $\alpha_s(M_Z)$ in the supersymmetric parameter space. We will find the

unification-scale threshold corrections that are necessary for gauge coupling unification with a given supersymmetric spectrum, and compare our results with the threshold corrections that arise in the minimal and missing-doublet SU(5) models. We will close with a similar analysis under the additional assumption of bottom-tau Yukawa unification.

2 Gauge Coupling Unification

We start by sketching our calculation of the \overline{DR} gauge parameters $\hat{g}_1(M_Z)$ and $\hat{g}_2(M_Z)$. As discussed above, we take as inputs the Fermi constant $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$, the Z -boson mass $M_Z = 91.187 \text{ GeV}$, the electromagnetic coupling $\alpha_{\text{EM}} = 1/137.036$, the top-quark mass m_t , and the supersymmetric parameters. From these we calculate the electromagnetic coupling $\hat{\alpha}$ and the weak mixing angle \hat{s}^2 in the \overline{DR} renormalization scheme,

$$\hat{\alpha} = \frac{\alpha_{\text{EM}}}{1 - \Delta\hat{\alpha}}, \quad \hat{s}^2 \hat{c}^2 = \frac{\pi \alpha_{\text{EM}}}{\sqrt{2} G_F M_Z^2 (1 - \Delta\hat{r})}, \quad (4)$$

where $\hat{c}^2 = \cos^2 \hat{\theta}(M_Z)$,

$$\begin{aligned} \Delta\hat{\alpha} = & 0.0685 \pm 0.0007 - \frac{\alpha_{\text{EM}}}{2\pi} \left\{ -\frac{7}{4} \log\left(\frac{M_W}{M_Z}\right) + \frac{16}{9} \log\left(\frac{m_t}{M_Z}\right) + \frac{1}{3} \log\left(\frac{m_{H^+}}{M_Z}\right) \right. \\ & \left. + \sum_{i=1}^6 \frac{4}{9} \log\left(\frac{m_{\tilde{u}_i}}{M_Z}\right) + \sum_{i=1}^6 \frac{1}{9} \log\left(\frac{m_{\tilde{d}_i}}{M_Z}\right) + \sum_{i=1}^3 \frac{1}{3} \log\left(\frac{m_{\tilde{e}_i}}{M_Z}\right) + \sum_{i=1}^2 \frac{4}{3} \log\left(\frac{m_{\chi_i^+}}{M_Z}\right) \right\}, \end{aligned} \quad (5)$$

and [3]

$$\Delta\hat{r} = \Delta\hat{\alpha} + \frac{\hat{\Pi}_W(0)}{M_W^2} - \frac{\hat{\Pi}_Z(M_Z)}{M_Z^2} + \text{vertex} + \text{box}. \quad (6)$$

Equation (5) includes the light quark contribution extracted from experimental data [4], together with the leptonic contribution. It also contains the logarithms of the W -boson, top-quark, charged-Higgs, squark, slepton, and chargino masses. In eq. (6), the $\hat{\Pi}$ denote the real and transverse parts of the gauge boson self-energies, evaluated in the \overline{DR} scheme. Equation (6) also includes the vertex and box contributions that renormalize the Fermi constant, as well as the leading higher-order m_t^4 and QCD standard-model corrections given in ref. [5]. (For a complete description of our calculation see ref. [6]. Note that in this letter, all hatted objects are \overline{DR} quantities, and all masses are pole masses.)

From these results we find the weak gauge couplings $\hat{g}_1(M_Z)$ and $\hat{g}_2(M_Z)$ using the \overline{DR} relations

$$\hat{g}_1(M_Z) = \sqrt{\frac{5}{3}} \frac{\hat{e}}{\hat{c}}, \quad \hat{g}_2(M_Z) = \frac{\hat{e}}{\hat{s}}, \quad (7)$$

where $\hat{\alpha} = \hat{e}^2/4\pi$. These couplings serve as the weak-scale boundary conditions for the two-loop supersymmetric RGE's. They implicitly determine the unification scale M_{GUT} through the condition $\hat{g}_1(M_{\text{GUT}}) = \hat{g}_2(M_{\text{GUT}}) \equiv g_{\text{GUT}}$.

We fix the strong coupling $\hat{g}_3(M_{\text{GUT}})$ at the scale M_{GUT} by the unification condition

$$\hat{g}_3(M_{\text{GUT}}) = g_{\text{GUT}}(1 + \epsilon_g) , \quad (8)$$

where ϵ_g parametrizes the model-dependent unification-scale gauge threshold correction. We then run $\hat{g}_3(M_{\text{GUT}})$ back to the Z -scale to find the \overline{MS} coupling

$$\alpha_s(M_Z) = \left(\frac{\hat{g}_3^2(M_Z)}{4\pi} \right) (1 - \Delta\alpha_s)^{-1} , \quad (9)$$

where $\Delta\alpha_s$ denotes the weak-scale threshold correction

$$\Delta\alpha_s = \frac{\hat{g}_3^2(M_Z)}{8\pi^2} \left(-\frac{1}{2} + \frac{2}{3} \log \left(\frac{m_t}{M_Z} \right) + \sum_{i=1}^{12} \frac{1}{6} \log \left(\frac{m_{\tilde{q}_i}}{M_Z} \right) + 2 \log \left(\frac{m_{\tilde{g}}}{M_Z} \right) \right) . \quad (10)$$

This procedure allows us to determine $\alpha_s(M_Z)$ for a fixed ϵ_g and a given supersymmetric spectrum. In what follows, we will make the additional assumption that the supersymmetric spectrum unifies as well. Therefore we also assume that the three gaugino masses unify to a common value of $M_{1/2} \equiv \hat{M}_{1/2}(M_{\text{GUT}})$, the scalar masses unify to a common scalar mass $M_0 \equiv \hat{M}_0(M_{\text{GUT}})$, and the soft trilinear scalar coupling parameters unify to $A_0 \equiv \hat{A}_0(M_{\text{GUT}})$.

We evolve the parameters $M_{1/2}$, M_0 and A_0 to the weak scale using the two-loop supersymmetric RGE's [7]. We require the parameters to be such that electroweak symmetry is spontaneously broken, as naturally occurs when the top-quark mass is large. Then the Higgs bosons H_1 and H_2 acquire expectation values $v_1 \equiv \hat{v}_1(M_Z)$ and $v_2 \equiv \hat{v}_2(M_Z)$; their ratio is denoted $\tan \beta \equiv \tan \hat{\beta}(M_Z) = v_2/v_1$.

We extract the supersymmetric masses from the running \overline{DR} parameters and $\tan \beta$ at the scale M_Z . (We choose $\mu > 0$, where the superpotential contains the term $+\mu\epsilon_{ij}H_1^iH_2^j$, with $\epsilon_{12} = +1$.) We then substitute these masses into eqs. (4) – (6), and repeat the entire procedure to find a self-consistent solution to the renormalization group equations. Typically, the process converges after only a few iterations. It allows us to predict $\alpha_s(M_Z)$ consistently, including all finite corrections.

As a point of reference, we show in Fig. 1 our results for $\alpha_s(M_Z)$ in the $M_0, M_{1/2}$ plane, in the absence of unification threshold corrections, for $m_t = 170$ GeV, $\tan \beta = 2$, and $A_0 = 0$. Comparing with the experimental value $\alpha_s(M_Z) = 0.117 \pm .005$ [8], we see that $\alpha_s(M_Z)$ is rather large.¹ If, for naturalness, we require the squark masses to be below 1 TeV, we obtain the lower bound

$$\alpha_s(M_Z) > 0.126 \quad (\text{No unification thresholds, } m_{\tilde{q}} \leq 1 \text{ TeV, } m_t = 170 \text{ GeV}),$$

assuming the validity of perturbation theory, which we take to mean that the \overline{DR} top-quark Yukawa coupling $\hat{\lambda}_t(M_{\text{GUT}}) \leq 3$. If we tighten this condition to $\hat{\lambda}_t(M_{\text{GUT}}) \leq 1$, the bound increases by 0.002. For smaller m_t , $\alpha_s(M_Z)$ is smaller. For example, if $m_t = 160$ GeV, the limit reduces by 0.002. These variations apply independently of the unification-scale thresholds. Our numbers agree qualitatively with the results of ref. [2].

¹The experimental error in $\Delta\hat{\alpha}$ leads to a ± 0.001 uncertainty in $\alpha_s(M_Z)$.

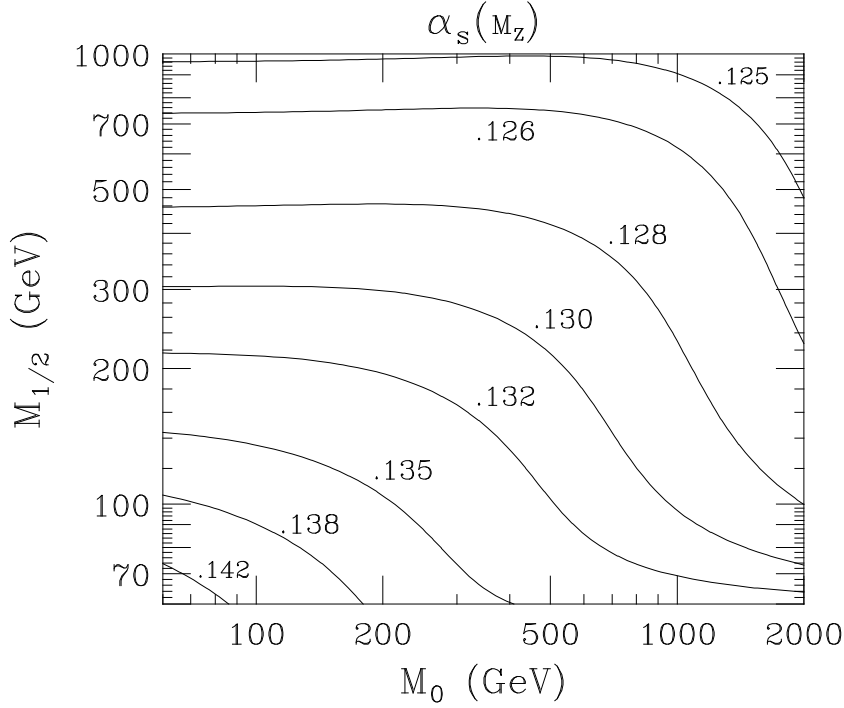


Figure 1: Contours of $\alpha_s(M_Z)$ in the $M_0, M_{1/2}$ plane with $m_t = 170$ GeV, $\tan \beta = 2$ and $A_0 = 0$. The region $m_{\tilde{q}} < 1$ TeV is bounded by $M_{1/2} \lesssim 400$ GeV and $M_0 < 1$ TeV.

The values of $\alpha_s(M_Z)$ quoted here are larger than in many previous analyses for two reasons. First, during the past few years the central value of s_{SM}^2 , as determined from precision electroweak measurements, has been decreasing. (This change is correlated with the increasing best-fit value of m_t .) A smaller value of s_{SM}^2 leads to an increase in $\alpha_s(M_Z)$, as can be seen from eq. (3). Second, the finite corrections decrease \hat{s}^2 , and therefore increase $\alpha_s(M_Z)$. The finite corrections are important in the region $M_{1/2} \lesssim 200$ GeV where $\alpha_s(M_Z)$ is appreciably larger than in the leading logarithmic approximation, as shown in Fig. 2.

Of course, the value of $\alpha_s(M_Z)$ can be reduced by a unification-scale threshold correction with $\epsilon_g < 0$. In the minimal SU(5) model [9], the unification-scale gauge threshold correction is [10]

$$\epsilon'_g = \frac{3g_{\text{GUT}}^2}{40\pi^2} \log \left(\frac{M_{H_3}}{M_{\text{GUT}}} \right), \quad (11)$$

where M_{H_3} is the mass of the color-triplet Higgs fermion that mediates nucleon decay.² From this expression, we see that $\epsilon'_g < 0$ whenever $M_{H_3} < M_{\text{GUT}}$. However, M_{H_3} is bounded from below by proton decay experiments, so $\epsilon'_g > 0$ throughout most of the parameter space.³

²The gauge threshold ϵ'_g also receives contributions from higher-dimensional Planck-scale operators that we ignore. They lead to a ± 0.006 uncertainty in $\alpha_s(M_Z)$ [11].

³We use the formulae given in ref. [12], and choose the conservative values $\beta = 0.003 \text{ GeV}^3$ and $|1 + y^{tK}| = 0.4$.

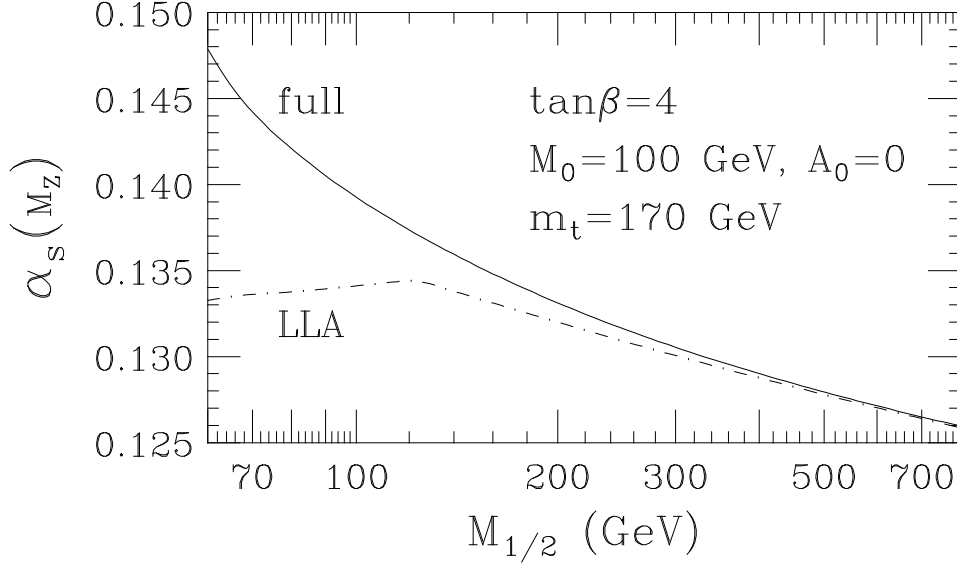


Figure 2: The \overline{MS} coupling $\alpha_s(M_Z)$ vs. $M_{1/2}$. The curve labeled LLA shows the result if we include only the logarithms of the supersymmetric masses (the leading logarithm approximation), while the solid line corresponds to the full result including all finite corrections.

In minimal SU(5) we find that the smallest possible values for $\alpha_s(M_Z)$ are typically even larger than those in Fig. 1. The only exception occurs for $M_0 \gg M_{1/2}$, where the proton decay amplitude is suppressed. This determines the lower limit

$$\alpha_s(M_Z) > 0.123 \quad (\text{Minimal SU(5), } m_{\tilde{q}} \leq 1 \text{ TeV, } m_t = 170 \text{ GeV}),$$

assuming $\hat{\lambda}_t(M_{\text{GUT}}) \leq 3$. In fact, as long as $m_{\tilde{q}} \leq 1 \text{ TeV}$, $\alpha_s(M_Z) < 0.127$ can only be obtained in the region $M_0 \simeq 1 \text{ TeV}$. For example, if $M_0 \leq 500 \text{ GeV}$, $\alpha_s(M_Z) > 0.128$.

The missing-doublet model is an alternative SU(5) theory in which the heavy color-triplet Higgs particles are split naturally from the light Higgs doublets [13]. In this model the unification-scale gauge threshold can be written as [14]

$$\epsilon_g'' = \frac{3g_{\text{GUT}}^2}{40\pi^2} \left\{ \log \left(\frac{M_{H_3}^{\text{eff}}}{M_{\text{GUT}}} \right) - \frac{25}{2} \log 5 + 15 \log 2 \right\} \simeq \epsilon_g' - 4\% . \quad (12)$$

Thus, for fixed M_{H_3} , the missing-doublet model has the same threshold correction as the minimal SU(5) model, minus 4%. In eq. (12), $M_{H_3}^{\text{eff}}$ is the effective mass that enters into the proton decay amplitude, so the bounds on M_{H_3} in the minimal SU(5) model also apply to $M_{H_3}^{\text{eff}}$ in the missing-doublet model.

The large negative correction in eq. (12) gives rise to much smaller values for $\alpha_s(M_Z)$. This is illustrated in Fig. 3, where we show the upper and lower bounds on ϵ_g in the minimal and missing-doublet SU(5) models, together with the values of ϵ_g necessary to obtain

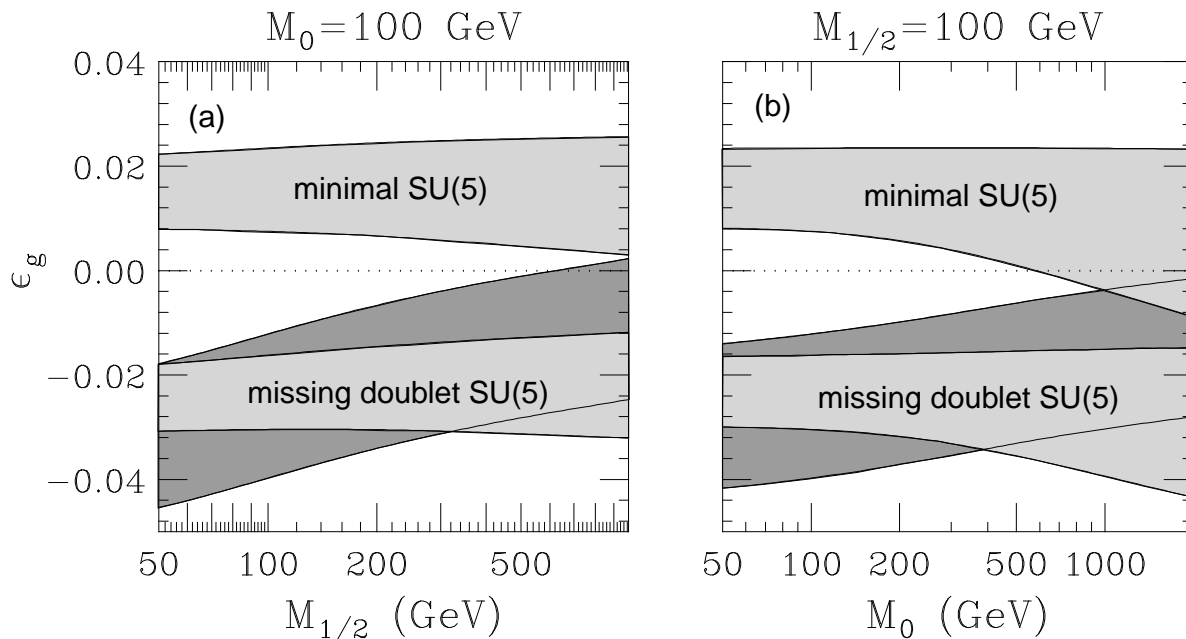


Figure 3: The light shaded regions indicate the allowed values of the gauge coupling threshold correction ϵ_g in the minimal and missing-doublet SU(5) models. The dark shaded region indicates the range of ϵ_g necessary to obtain $\alpha_s(M_Z) = 0.117 \pm 0.1$.

$\alpha_s(M_Z) = 0.117 \pm 0.01$. For both SU(5) models we bound ϵ_g from below by the limits on proton decay and from above by the requirement $M_{H_3} < 10^{19}$ GeV. Note that for the missing-doublet model, the region of allowed ϵ_g nearly overlaps the region with $\alpha_s(M_Z) = 0.117 \pm 0.01$, regardless of the supersymmetric particle masses.

3 Yukawa Coupling Unification

In typical SU(5) models the bottom and tau Yukawa couplings unify at the scale M_{GUT} . In the remainder of this letter, we will investigate bottom-tau unification to the same next-to-leading-order accuracy that we used in our analysis of the gauge couplings. Our analysis is based on two-loop RGE's and on the complete one-loop expressions that relate the \overline{DR} Yukawa couplings to the physical fermion masses [6].

Let us illustrate our procedure for the case of the tau Yukawa coupling. We start by determining the \overline{DR} expectation value \hat{v} from the Z -boson mass,

$$\frac{1}{4}(\hat{g}'^2 + \hat{g}^2)\hat{v}^2 = M_Z^2 + \hat{\Pi}_Z(M_Z^2). \quad (13)$$

Then, given the pole mass $m_\tau = 1.777$ GeV [8], we find the tau Yukawa coupling $\hat{\lambda}_\tau(M_Z)$ using the \overline{DR} relation

$$\hat{\lambda}_\tau(M_Z) \hat{v} \cos \beta / \sqrt{2} = m_\tau + \hat{\Sigma}_\tau(m_\tau), \quad (14)$$

where $\hat{\Sigma}_\tau(m_\tau) = \Sigma_1 + m_\tau \Sigma_\gamma$, and the tau self-energy is $\Sigma_1 + \not{p} \Sigma_\gamma + \gamma_5 (\Sigma_5 + \not{p} \Sigma_{\gamma_5})$. We follow a similar procedure to find the top-quark Yukawa coupling $\hat{\lambda}_t(M_Z)$.

Once we have the top and tau Yukawa couplings at the scale M_Z , we evolve them to the scale M_{GUT} using the two-loop supersymmetric RGE's [7]. At M_{GUT} we fix the \overline{DR} bottom-quark Yukawa coupling $\hat{\lambda}_b(M_{\text{GUT}})$ through the unification condition

$$\hat{\lambda}_b(M_{\text{GUT}}) = \hat{\lambda}_\tau(M_{\text{GUT}})(1 + \epsilon_b), \quad (15)$$

where ϵ_b parametrizes the unification-scale Yukawa threshold correction. We then run all the Yukawa couplings back to the Z scale, and iterate the procedure self-consistently to determine $\hat{\lambda}_b(M_Z)$. Finally, we apply the weak-scale threshold corrections to find the pole mass for the bottom quark.

For a top-quark pole mass $m_t = 170$ GeV, this procedure typically gives a bottom-quark mass outside the range of experiment (which we take to be $4.7 < m_b < 5.2$ GeV [8]). From previous analyses we know that for $m_t < 200$ GeV there are two regions of $\tan\beta$ where bottom-tau unification might occur: $\tan\beta \lesssim 2$ and $\tan\beta \gtrsim 40$. Here we focus on the small $\tan\beta$ branch, where the top-quark Yukawa coupling at the unification scale is large, $\hat{\lambda}_t(M_{\text{GUT}}) \gtrsim 1$.

As a point of reference, we first present our results with no unification-scale threshold corrections. In Fig. 4 we show m_b and $\alpha_s(M_Z)$ versus m_t , for various values of $\tan\beta$, M_0 , and $M_{1/2}$, with $\hat{\lambda}_t(M_{\text{GUT}}) = 3$. From the figure we see that the bottom-quark pole mass is generally too large, unless the squark masses are of order 1 TeV.

The value $\hat{\lambda}_t(M_{\text{GUT}}) = 3$ was chosen because it lies on the edge of perturbation theory. Smaller values of $\hat{\lambda}_t(M_{\text{GUT}})$ give rise to larger values of m_b , so the curves in Fig. 4 can be interpreted as lower limits on the bottom-quark mass. If we require the squark masses to be below 1 TeV, we obtain the lower bound

$$m_b > 5.1 \text{ GeV} \quad (\text{No unification thresholds, } m_{\tilde{q}} \leq 1 \text{ TeV, } m_t = 170 \text{ GeV}).$$

In the absence of unification-scale threshold corrections, we see that, in the small $\tan\beta$ region, bottom-tau Yukawa unification favors large values of m_b and large values of the supersymmetric mass scale. The above bound depends slightly on the top-quark mass, and sensitively on $\hat{\lambda}_t(M_{\text{GUT}})$. For $m_t = 160$ GeV, the limit reduces by 0.1 GeV. The bound increases by 0.3 GeV for $\hat{\lambda}_t(M_{\text{GUT}}) = 2$ and by 0.8 GeV for $\hat{\lambda}_t(M_{\text{GUT}}) = 1$. These variations also apply for $\epsilon_b \neq 0$.

As with $\alpha_s(M_Z)$, the picture is altered by unification-scale threshold corrections. To understand their effects, we first note the striking similarity between the m_b and $\alpha_s(M_Z)$ curves in Fig. 4, which implies that the value of m_b is tightly correlated with the value of $\alpha_s(M_Z)$. This leads us to expect that the gauge threshold correction will have an important effect on m_b .

In Fig. 5 we show the prediction for m_b versus m_t , for fixed $\epsilon_b = 0$, with $\hat{\lambda}_t(M_{\text{GUT}}) = 2, 3$ and ϵ_g chosen to give a fixed value of $\alpha_s(M_Z)$. We see that any model which gives $\alpha_s(M_Z) \simeq$

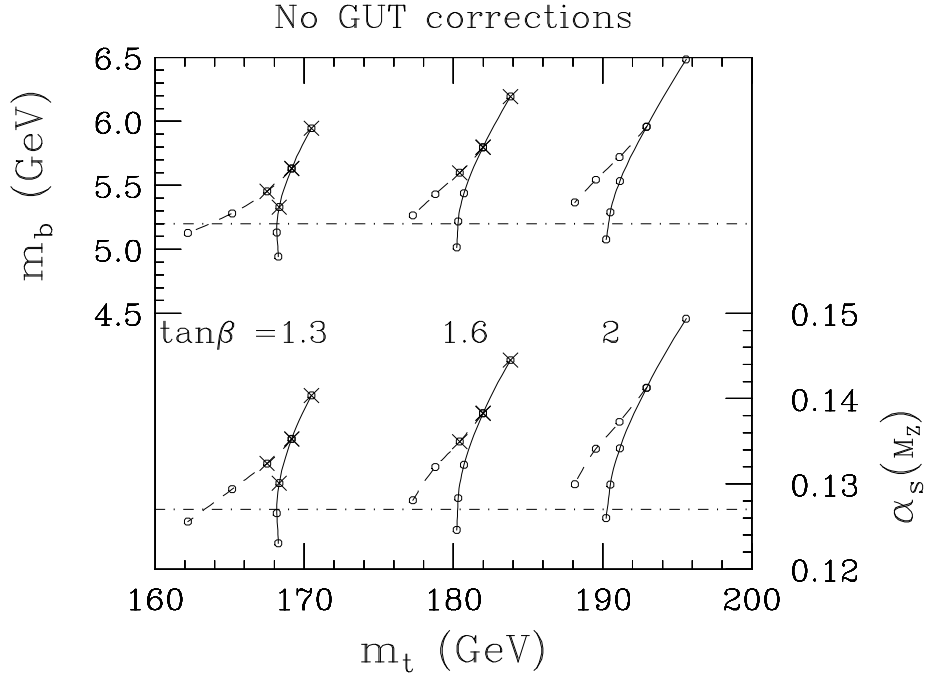


Figure 4: The bottom-quark mass and $\alpha_s(M_Z)$ vs. m_t for the case of no unification-scale thresholds, for various values of $\tan\beta$, with $A_0 = 0$ and $\hat{\lambda}_t(M_{\text{GUT}}) = 3$. The right (solid) leg in each pair of lines corresponds to $M_{1/2}$ varying from 60 to 1000 GeV, with M_0 fixed at 60 GeV. The left (dashed) leg corresponds to M_0 varying from 60 to 1000 GeV, with $M_{1/2} = 100$ GeV. On the solid lines the circles mark, from top to bottom, $M_{1/2} = 60, 100, 200, 400$, and 1000 GeV, and on the dashed lines the circles mark $M_0 = 60, 200, 400$, and 1000 GeV. Note that the lowest point on each left leg and the second-to-lowest point on each right leg corresponds to $m_{\tilde{q}} \simeq 1$ TeV. The horizontal dashed lines indicate $m_b = 5.2$ GeV and $\alpha_s(M_Z) = 0.127$. The \times 's mark points with one-loop Higgs mass $m_h < 60$ GeV.

0.12 also predicts $m_b \simeq 5$ GeV, with $\epsilon_b = 0$. Furthermore, any $-23\% < \epsilon_b < 9\%$ gives an acceptable value for m_b in this case. Alternatively, if $\alpha_s(M_Z) \simeq 0.13$, a large and negative ϵ_b is required to achieve the central value for m_b , namely $-33\% < \epsilon_b < -8\%$. Figure 5 shows that these conclusions hold independently of the top-quark mass and the supersymmetric mass scale.

In Fig. 6 we plot the most favorable values of $\alpha_s(M_Z)$ and m_b in the minimal SU(5) model. In this model the Yukawa threshold is given by [15]

$$\epsilon'_b = \frac{1}{16\pi^2} \left[4g_{\text{GUT}}^2 \left(\log \left(\frac{M_V}{M_{\text{GUT}}} \right) - \frac{1}{2} \right) - \hat{\lambda}_t^2(M_{\text{GUT}}) \left(\log \left(\frac{M_{H_3}}{M_{\text{GUT}}} \right) - \frac{1}{2} \right) \right], \quad (16)$$

where M_V is the mass of the superheavy SU(5) gauge bosons. We define the most favorable value as follows. We first minimize the value of $\alpha_s(M_Z)$ by picking the smallest possible M_{H_3} consistent with nucleon decay. We then minimize m_b by choosing the smallest M_V allowed by the constraint that the SU(5) model Yukawa couplings remain perturbative, $M_V > \max(0.3M_{H_3}, 0.5M_{\text{GUT}})$. From the figure we see that this brings m_b to an acceptable

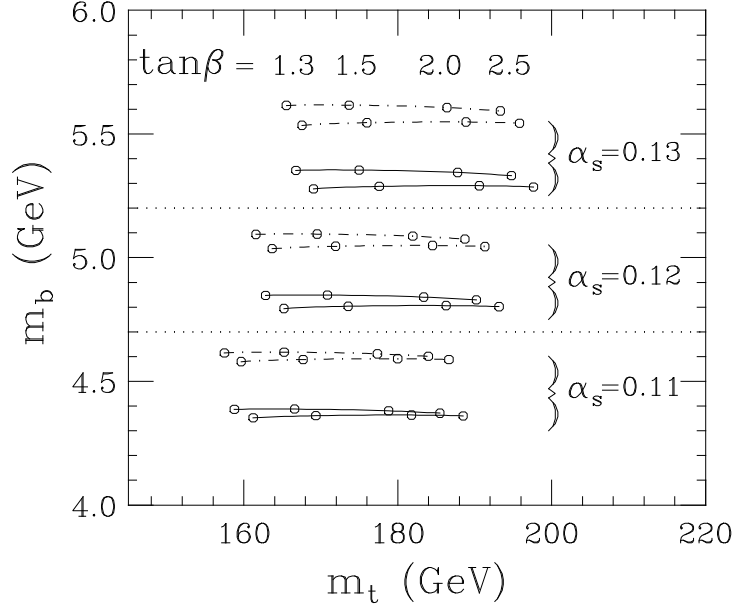


Figure 5: The bottom-quark mass vs. the top-quark mass for fixed values of $\alpha_s(M_Z)$ and various $\tan\beta$. The solid lines correspond to $\hat{\lambda}_t(M_{\text{GUT}}) = 3$ while the dashed lines correspond to $\hat{\lambda}_t(M_{\text{GUT}}) = 2$. The upper line in each pair corresponds to a light supersymmetric spectrum with $M_0 = M_{1/2} = 80$ GeV. The lower line in each pair corresponds to a heavy spectrum, $M_0 = 1000$ GeV, $M_{1/2} = 500$ GeV.

range for squark masses of order 1 TeV. Indeed, we find the limit⁴

$$m_b > 5.1 \text{ GeV} \quad (\text{Minimal SU(5), } m_{\tilde{q}} \leq 1 \text{ TeV, } m_t = 170 \text{ GeV}). \quad (17)$$

In contrast to minimal SU(5), the missing-doublet model can readily accommodate bottom-quark masses in the range $4.7 \text{ GeV} < m_b < 5.2 \text{ GeV}$, with $\epsilon_b \simeq 0$. For example, with $m_t = 170$ GeV and $M_0 = M_{1/2} = 100$ GeV, minimal SU(5) requires ϵ_b in the range -15 to -65% . The missing-doublet model, however, requires ϵ_b in the range -30 to $+20\%$. Hence even for small $\mathcal{O}(100 \text{ GeV})$ supersymmetric masses the missing-doublet model naturally accommodates both gauge and Yukawa coupling unification.

4 Conclusion

In this paper we have computed the complete one-loop weak-scale threshold corrections in the minimal supersymmetric standard model. We used them to study gauge and Yukawa unification with and without unification-scale threshold corrections. In the absence of such corrections we find that $\alpha_s(M_Z)$ and m_b are large unless the squark masses are larger than

⁴We could try to decrease m_b further by increasing M_{H_3} , which decreases ϵ'_b . However, the corresponding increase in $\alpha_s(M_Z)$ compensates for this so that the limit in eq. (17) remains unchanged.

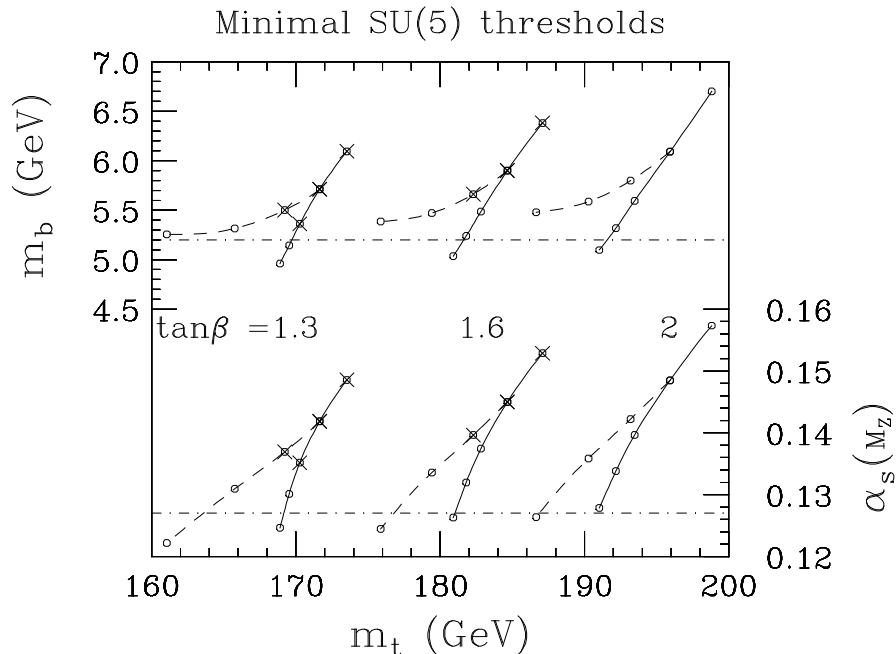


Figure 6: The same as Fig. 4, with the most favorable minimal SU(5) threshold corrections, as defined in the text.

about 1 TeV. Adding minimal SU(5) threshold corrections, and requiring $m_{\tilde{q}} \leq 1$ TeV, we find that $\alpha_s(M_Z) < 0.127$ occurs only for $M_{1/2} \ll M_0$ and $M_0 \simeq 1$ TeV. Additionally, the condition $m_b < 5.2$ GeV is fulfilled only for $m_{\tilde{q}} \gtrsim 1$ TeV. In the missing-doublet model, however, the threshold corrections permit acceptable values for $\alpha_s(M_Z)$ and m_b for either a light or heavy supersymmetric spectrum.

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